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Simulation of Birth-Death Dynamics in Time-Variant Stochastic Radio Channels

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Abstract—We consider an analytically tractable class of time-variant stochastic radio channel models. All models of this class are designed such that individual multipath components emerge and vanish according to a temporal birth-death process. The birth-death random process is governed by two facilitating assumptions: *i*) stationary emergence times, and *ii*) i.i.d. lifetimes. Multipath channel models with such temporal birth-death dynamics have appeared in the literature several times. More specifically, such channel models have appeared with assumption *i*) specialized to that of a Poisson point process and with assumption *ii*) specialized to that of i.i.d. exponentials. So far, these two special-case assumptions have been invoked by default without justification or clarification for their necessity. Here, we establish and justify their indispensable necessity from a simulation practical point of view. Specifically, we obtain a tractable and exact (non-approximate) Markovian simulation recipe for drawing realizations of time-variant stochastic radio channels with temporal birth-death dynamics.

I. INTRODUCTION AND PRELIMINARIES

A simple, flexible, and commonly used stochastic model for time-variant channel transfer functions is given by [1]

$$H(t, f) = \sum_{\ell=1}^{L(t)} \alpha_{\ell}(t) \exp(-j2\pi f \tau_{\ell}(t)). \quad (1)$$

The integer-valued random variable $L(t)$ gives the instantaneous number of path components at time t , $\alpha_{\ell}(t)$ is the random complex-valued gain of the ℓ 'th path, and $\tau_{\ell}(t)$ is the associated random propagation delay. Time- and space-varying multipath propagation phenomena, e.g. path components which emerge and vanish, occur partially due to the movements of transmitter, receiver, and surrounding scatterers [1]. Transitions of the random process $L(\cdot)$ reflect when different path components emerge and vanish. The random process $L(\cdot)$ can be generated in numerous ways, for instance according to the following tractable assumptions:

- i*) **Stationary emergences:** The collection of time instances where new path components emerge forms a stationary point process on the real line.
- ii*) **i.i.d. lifetimes:** The non-negative lifetimes (or periods) of individual path components are i.i.d.

In [2] we show how the assumptions *i*) and *ii*) can be conveniently incorporated using an approach based on *spatial point processes*¹ [3]. Specifically, denote by Y the *one-dimensional* point process from *i*) and denote by $\{p_y : y \in Y\}$ the

¹The spatial point process approach used in [2] serves essentially to bypass the enumeration problem which occurs since every transition of $L(\cdot)$ leaves the need for a cumbersome re-ordering and bookkeeping of indices in (1).

collection of *non-negative* periods from *ii*). The subscript y on each period p_y serves as an identifier for its underlying point. By construction, the random collection $\{(y, p_y) : y \in Y\}$ is a *marked point process* [3] with (stationary) points in \mathbb{R} and (i.i.d.) marks in \mathbb{R}_+ . Alternatively, this marked point process can be viewed as a *two-dimensional* point process

$$X := \{(y, p_y) : y \in Y\}, \quad (2)$$

i.e. as an *unmarked* point process with points in $\mathbb{R} \times \mathbb{R}_+$. Then, each two-dimensional point $\mathbf{x} = (y, p) \in X$ has components y and p interpreted as “birth time” and “lifetime”, respectively, and a subscript identifier on p is no longer needed.

In terms of the two-dimensional point process X , the channel model in (1) can now be reformulated as [2]

$$H(t, f) = \sum_{\mathbf{x} \in X} \mathbb{1}[\mathbf{x} \in B_t] \alpha_{\mathbf{x}}(t) e^{-j2\pi f \tau_{\mathbf{x}}(t)}, \quad (3)$$

$$L(t) = |X \cap B_t| = \sum_{\mathbf{x} \in X} \mathbb{1}[\mathbf{x} \in B_t], \quad t \in \mathbb{R}, \quad (4)$$

where $|\cdot|$ denotes set cardinality while $\mathbb{1}[\cdot]$ denotes a generic *indicator function* taking value one if the logical statement in brackets is fulfilled and zero otherwise. The time-indexed quantity B_t in (3) and (4) is the triangular-shaped region

$$B_t := \{(\tilde{y}, \tilde{p}) : \tilde{y} < t, \tilde{y} + \tilde{p} > t\} \subset \mathbb{R} \times \mathbb{R}_+. \quad (5)$$

The relationship in (4) states that $L(t)$ is equal to the random number of points from X falling in the region B_t , see Fig. 1. An arbitrary point $\mathbf{x} = (y, p) \in X$ contributes to the value of the sum in (4) only if it *emerges before* time t (i.e. $y < t$) and *vanishes after* time t (i.e. $y + p > t$). As a consequence of *i*) and *ii*), the integer-valued continuous-time random process $L(\cdot)$ is strict-sense stationary [2], [4].

Notice how the collections $\{\alpha_{\ell}(\cdot)\}$ and $\{\tau_{\ell}(\cdot)\}$ of random processes from (1) have been replaced by equivalent collections $\{\alpha_{\mathbf{x}}(\cdot)\}$ and $\{\tau_{\mathbf{x}}(\cdot)\}$ in (3). These two new collections are indexed using the points from X as proposed in [2]. The representation in (3) inherits several analytical benefits compared to the traditional representation in (1), especially in terms of the ability to track individual path components in time, see Fig. 1.

In the literature [5]–[8], assumptions *i*) and *ii*) were originally introduced in more restricted versions:

- i*)[†] Special-case of *i*): Poisson point process [9].
- ii*)[†] Special-case of *ii*): Exponential distribution.

In the remaining part of this paper we restrict to the special-case assumptions *i*)[†] and *ii*)[†]. In the earliest channel modeling

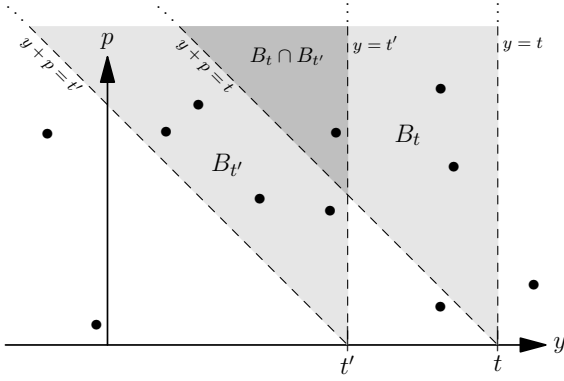


Figure 1. Each black bullet represents a point from the two-dimensional point process X . Points falling within the region B_t indicate path components which are *active* in the radio channel (1) at time t . Those falling in $B_t \cap B_{t'}$ are contributing to the channel at time instances t' , t , and in-between.

contribution by S. J. Papantoniou [5], the construct via $i)^\dagger$ and $ii)^\dagger$ is identified² as an $M/M/\infty$ queue. Accordingly, all known properties of this particular queue apply directly to the random process $L(\cdot)$. For instance, $L(t)$ is Poisson distributed for each fixed $t \in \mathbb{R}$ [4, Sec. 16-2]. The original motivation for using $i)^\dagger$ and $ii)^\dagger$ is (quoting Papantoniou [5, Sec. 2.2.6]): “that these assumptions endow the model with *simple mathematics*”.

Later contributions such as [7] and [8] have seemingly employed the special-case assumptions $i)^\dagger$ and $ii)^\dagger$ by default. In addition, [8] proposes cumbersome implementation guidelines for computer simulation, e.g. a heuristic channel initialization scheme as well as a procedure for approximating the continuous-time process $L(\cdot)$ on a discrete sampling grid. Yet, both of these approximate simulation guidelines can be circumvented and substituted by exact procedures upon taking direct advantage of the facilitating aspects of the point process perspective in (4). As opposed to [8], the earlier contributions [5]–[7] entirely omit discussions related to computer simulation of their respective (birth-death) channel models.

This paper presents three main contributions. Firstly, we justify and clarify Papantoniou’s claim from the point of view of computer simulation. As shown in [2], $i)^\dagger$ and $ii)^\dagger$ are indeed analytically tractable in general, but as shown here they prove as well highly facilitating for simulation purposes. Secondly, we show the utility of the point process view in (4) with respect to the actual implementation of the temporal birth-death $L(\cdot)$, especially in the context of radio channel modeling. Knowing that $L(\cdot)$ can be seen as an $M/M/\infty$ queue is not equally advantageous since this perspective does not aid in being able to track individual path components in (1). Thirdly, we show how the memoryless property of the exponential distribution can be exploited to represent the continuous-time birth-death process $L(\cdot)$ on an arbitrary discrete sampling grid (which is needed in practice). Compared to [8, Sec. III-C] our representation is exact, i.e. it does not rely on approximations such as disregarding “tiny” probabilities of multiple jump events within “tiny” intervals of time.

²More precisely, [5] presents a *space-varying* approach which simplifies to a time-varying model like (1) and (3) upon assuming a receiver trajectory with constant velocity vector.

II. SIMULATION OF THE BIRTH-DEATH PROCESS $L(\cdot)$ IN TIME-VARYING RADIO CHANNELS

Under the special-case assumptions $i)^\dagger$ and $ii)^\dagger$ there are several (equivalent) ways to view the birth-death process $L(\cdot)$. The process can be seen as an $M/M/\infty$ queue, as a continuous-time Markov chain [10, Sec. 7.4], as generalized shot-noise, but it can also be seen via the point process perspective in (4). Specifically, it can be seen as the “time-sliding” region count displayed in Fig. 1. All of the aforementioned views have their individual advantages and drawbacks.

In queuing theory it is usually the queue itself which is of primary interest, not the individual customers (they just temporally alter the length of the queue). When considering the channel model in (1) and (3) the situation is different. It makes a crucial difference if we need to be able to track the individual path components over time. In radio channel characterization we often wish to “correlate” the channel with itself at different time-frequency instances. Hence, it is important to be able to identify and track if path components are still present, if new ones have emerged, or if some have vanished in-between any two time instances t' and t . In Fig. 1 we have $L(t') = 5$ and $L(t) = 3$ but only one component is shared. Accordingly, the considered point process perspective is beneficial for channel modeling purposes.

A. Notation and Properties of Poisson Point Processes

Under assumption $i)^\dagger$ the random collection Y in (2) is a stationary Poisson point process with constant *intensity function* $\varrho_Y(\cdot)$. Thus, $\varrho_Y(y) = \lambda_e$ for all $y \in \mathbb{R}$, for some positive constant λ_e (subscript abbreviating “emerge”).

By $ii)^\dagger$ the collection of periods $\{p_y : y \in Y\}$ is such that

$$p_y \stackrel{\text{i.i.d.}}{\sim} f_{\text{period}}(\cdot), \quad f_{\text{period}}(p) = \mathbb{1}[p \geq 0] \lambda_v \exp(-\lambda_v p),$$

for some positive rate parameter λ_v (subscript abbreviating “vanish”). Since the periods are mutually independent it follows that the two-dimensional point process X in (2) is also a Poisson point process (by the *Marking Theorem* for Poisson point processes [9, Sec. 5.2]). The Poisson point process X is *inhomogeneous* with intensity function given by [9, Sec. 5.2]

$$\varrho_X(\mathbf{x}) = \varrho_X(y, p) = \varrho_Y(y) f_{\text{period}}(p) = \lambda_e \lambda_v \exp(-\lambda_v p),$$

i.e. this intensity function is constant with y and decays exponentially with p . By the equality in (4) and the fact that X is a Poisson point process, it follows immediately that $L(t)$ is a Poisson distributed random variable for any fixed $t \in \mathbb{R}$. The mean of $L(t)$ is obtained by integrating the intensity function $\varrho_X(\cdot)$ across the region B_t , i.e.

$$\mathbb{E}[L(t)] = \mathbb{E}\left[\sum_{\mathbf{x} \in X} \mathbb{1}[\mathbf{x} \in B_t]\right] = \int_{B_t} \varrho_X(\mathbf{x}) d\mathbf{x} = \frac{\lambda_e}{\lambda_v},$$

which does not depend on time t , in accordance with $L(\cdot)$ being strict-sense stationary. The property of $L(t)$ being Poisson distributed with mean λ_e/λ_v was obtained by Papantoniou [5] using arguments from queuing theory. In [2], this property is readily obtained as a result of the point process perspective.

B. Initialization of $L(\cdot)$ at Time t'

Suppose we wish to initialize the channel in (1) at some time instance $t' \in \mathbb{R}$, e.g. $t' = 0$. In particular, we then have to initialize the temporal birth-death process $L(\cdot)$ at this particular time instance. Since $L(t')$ follows a Poisson distribution with mean λ_e/λ_v we can indeed generate $L(t')$ accordingly. Conditioned on $L(t')$, the points

$$\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{L(t')}\} = X \cap B_{t'}, \quad \mathbf{x}_\ell = (y_\ell, p_\ell), \quad (6)$$

should be drawn i.i.d. according to the (conditional) joint pdf

$$\begin{aligned} f(y, p; t') &= \frac{\mathbb{1}[(y, p) \in B_{t'}] \varrho_x(y, p)}{\int_{B_{t'}} \varrho_x(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}} \\ &= \mathbb{1}[0 < t' - y < p] \lambda_v^2 \exp(-\lambda_v p), \end{aligned} \quad (7)$$

i.e. according to a *truncated* and *normalized* version of the intensity function $\varrho_x(\cdot)$ [3], [9]. We stress the fact that the integer-labeling employed in (6) does not indicate any ordering of the points whatsoever. Notice also that the set $X \cap B_{t'}$ in (6) can potentially be empty since $L(t')$ was drawn from a Poisson distribution. The individual components y_ℓ and p_ℓ of the two-dimensional point \mathbf{x}_ℓ are obviously dependent due to the triangular shape of the region $B_{t'}$. The intuitive argument is of course that we have conditioned on the points in (6) to be “active” at time t' (i.e. they have not yet vanished).

By change of variables and by marginalizing the (conditional) joint pdf in (7) we readily find that

$$t' - y_\ell \sim \text{Exp}(\lambda_v), \quad y_\ell + p_\ell - t' \sim \text{Exp}(\lambda_v), \quad (8)$$

i.e. the lifespan which has already elapsed (from the past) and the lifespan which remains (in the future) are both exponentially distributed, *and in fact they are independent!* Thus, the (conditional) marginal distribution of the lifetime p_ℓ is not exponential, rather its distribution is that of the sum of the two independent exponentials in (8) and hence $p_\ell \sim \Gamma(2, \lambda_v)$, namely a gamma distribution³. This fact can also be verified by direct marginalization in (7). Conditioned on p_ℓ , the corresponding emergence time y_ℓ has a uniform distribution (into the past), i.e. $y_\ell | p_\ell \sim \mathcal{U}(t' - p_\ell, t')$. This is not surprising since by *ii*)[†], the collection of emergence times originates from a stationary Poisson point process.

The above procedure describes how to correctly initialize the non-negative integer $L(t')$ together with the individual components in (6). For comparison, [8] always initializes $L(t') = 0$ followed by a temporal “burn-in” to allow the birth-death process to evolve and stabilize before running the actual simulation. The procedure outlined in this paper allows for instantaneous and exact initialization of the channel in (3).

C. Temporal Evolution of $L(\cdot)$ in the Interval $[t', t'']$

Suppose that the birth-death process $L(\cdot)$ has been initialized at time t' in a state of equilibrium as described in the previous subsection. We can now arbitrarily select a stopping

³Compare this “conditional property” to that of the lifetime of a newly emerged component. By *ii*)[†], the lifetime of a newly emerged component should be assigned from an exponential distribution.

time $t'' > t'$ and then generate a realization of X restricted to the unbounded rectangular strip $F(t', t'') := [t', t''] \times \mathbb{R}_+$, see Fig. 2. To generate a realization of X restricted within $F(t', t'')$ we draw a Poisson distributed number with mean $\mathbb{E}[|X \cap F(t', t'')|] = \lambda_e(t'' - t')$, and then we distribute this amount of points inside $F(t', t'')$ according to i.i.d. draws from a truncated and normalized version of the intensity function $\varrho_x(\cdot)$. Essentially, this means that we need to generate pairs of uniformly and exponentially distributed random variables (all mutually independent). To calculate the realization of the temporal birth-death process $L(\cdot)$, we “slide” the triangular region B_t in (5) from $t = t'$ until $t = t''$, see Fig. 2.

The procedure above is suitable if we know in advance the necessary duration of our simulation. Lengthy simulations require in general a vast amount of numbers to be stored in advance. However, due to Markovian properties of $L(\cdot)$, there is an interesting alternative to the above simulation procedure.

D. Markovian Temporal Evolution of $L(\cdot)$ in $[t', \infty)$

Suppose that the birth-death process $L(\cdot)$ has been initialized at time t' such that $L(t')$ has been drawn from a Poisson distribution. Now we do not explicitly generate the individual points in (6) anymore. Instead, we make use of the property in (8), namely that the remaining lifespan $y_\ell + p_\ell - t'$ of each path component has an exponential distribution. What occurred in the past is no longer relevant, i.e. we are not interested in knowing when individual path components emerged. In fact, we are now concerned only with the next transition of $L(\cdot)$ which occurs sometime in the future. *We then maintain this concern one single transition at a time.* There are only two possibilities for the next transition since the point process X in (2) has *almost surely* no repetitions of points. Either a new path component emerges or a single of the existing ones vanishes. Thus, the birth-death process $L(\cdot)$ experiences a random “*increment*” from the set $\{-1, +1\}$ at a random time instance in the future. Once this increment has been assigned we wait yet another random time instance until the next increment from $\{-1, +1\}$ arrives, and so on.

If these random “*waiting times*” are always exponentially distributed it means that we are essentially dealing with a continuous-time Markov chain. This is indeed the case. After initialization at time t' , the first transition of $L(\cdot)$ occurs either when a new path component emerges (after a random time E), or when one of the existing components vanish (their individually remaining lifetimes can conveniently be denoted by $V_1, V_2, \dots, V_{L(t')}$). Accordingly, after initialization at time t' , the first transition of $L(\cdot)$ occurs at time $t' + \min\{V_1, V_2, \dots, V_{L(t')}, E\}$, where

$$V_\ell \sim \text{Exp}(\lambda_v), \quad \ell = 1, 2, \dots, L(t'), \quad E \sim \text{Exp}(\lambda_e). \quad (9)$$

The $L(t') + 1$ random variables in (9) are mutually independent and it is well-known that the minimum of a fixed number of independent exponentials has an exponential distribution [10, Sec. 3.10.1]. Thus, conditioned on $L(t)$ and now for arbitrary t , we conveniently define a waiting-time random variable

$$T_{L(t)} := \min\{V_1, V_2, \dots, V_{L(t)}, E\} \sim \text{Exp}(L(t)\lambda_v + \lambda_e)$$

as well as the indicator $I_{L(t)} := \mathbb{1}[E < \min\{V_1, \dots, V_{L(t)}\}]$. Then one can readily verify that

$$\Pr(I_{L(t)} = 1) = \Pr(E < \min\{V_\ell\}) = \frac{\lambda_e}{L(t)\lambda_v + \lambda_e},$$

and in fact, $T_{L(t)}$ and $I_{L(t)}$ are *independent!* Hence, we generate the random variable $T_{L(t)}$ with its distribution depending on $L(t)$. The associated increment from $\{-1, 1\}$ is independently determined from the realization of $I_{L(t)}$. The larger the value of $L(t)$ the smaller the expected time $\mathbb{E}[T_{L(t)}]$ until transition. Additionally, the more components currently attending in the channel, the more likely it is that one of the existing path components will soon vanish. In case the increment is minus one, i.e. $I_{L(t)} = 0$, then we simply remove an arbitrary component uniformly at random (since $V_1, V_2, \dots, V_{L(t)}$ are i.i.d.). Overall, instead of simulating $L(t) + 1$ random variables we can always do with simulating only two random variables. Thus, we sequentially generate transition times and increments of the process $L(\cdot)$.

In practical simulation studies we often need to represent the channel in (3) on a grid discretized in time and in frequency. Suppose we want to simulate the temporal birth-death process $L(\cdot)$ on some regular or irregular sampling grid $(t_n : n \in \mathbb{N}_0)$ with $t_0 = t'$ (initialization time) and $t_n < t_{n+1}$ for all n . Doing so yields the sequence

$$L(t_0), L(t_1), \dots, L(t_n), \dots, \quad (10)$$

and in case of a regular sampling grid, the time-step parameter $t_{n+1} - t_n = \Delta t$ could for instance be dictated by the *signalling* or the *sampling* period of a particular communication system (e.g. OFDM-based). Obviously, the transitions of $L(\cdot)$ occur in continuous time and not on the discrete sampling grid defined above. However, by the memoryless property of the exponential distribution, we can simply “reset the clock” as long as “nothing has yet happened”. Pseudo-code instructions read as follows:

Initialization:

Define $\lambda_e > 0$, $\lambda_v > 0$, and t_n for all n ; (parameters)

$L \sim \text{Poisson}(\lambda_e/\lambda_v)$;

$t = t_0$; $L(t) = L$; (assign initial count)

Temporal evolution across the time-grid:

for $n = 0, 2, 3, \dots$

$T_{\text{cumul}} = 0$; (reset sum variable)

while $(T_{\text{cumul}} < t_{n+1} - t_n)$ { (trivially satisfied at first)

$T \sim \text{Exp}(L\lambda_v + \lambda_e)$; (time until next event)

$T_{\text{cumul}} = T_{\text{cumul}} + T$; (time accumulator)

if $(T_{\text{cumul}} > t_{n+1} - t_n)$ {break;} (exit while-loop)

else { (revisit while-loop)

$U \sim \mathcal{U}(0, 1)$; (a probability)

if $(U < \frac{\lambda_e}{L\lambda_v + \lambda_e})$ { $L = L + 1$;} (new emergence)

else { $L = L - 1$;} (one component vanished)

} (revisit while-loop)

end while;

$t = t_{n+1}$; (move one time-step ahead)

$L(t) = L$; (assign count)

end for;

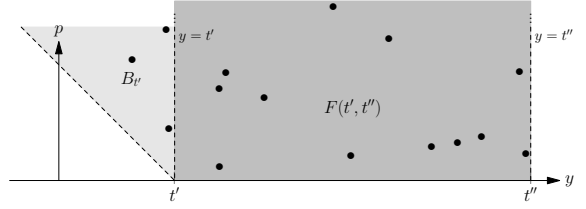


Figure 2. The unbounded rectangular strip $F(t', t'')$ from Sec. II-C.

The while-loop in the above pseudo-code is present to account for the fact that multiple transition events can occur between two consecutive sampling points of the time-grid. The approximate approach in [8] is based on picking a time-step parameter Δt sufficiently small so that within each time-step the probability of multiple transition events can be neglected in practice. The approach in this paper is exact (non-approximate) and valid for any sampling grid, regular or irregular. Hence, the simulation procedure described here can also be used for block-wise burst communications with long empty (or silent) gaps in-between consecutive transmission bursts.

E. Remarks on Generalization Attempts

The key feature of assumption $i)^\dagger$ is that the distribution of $L(t) = |X \cap B_t|$ is known to be Poisson, even without assumption $ii)^\dagger$ (the so-called $M/G/\infty$ queue). In this case we still know how to draw and distribute the points in (6), namely since these points belong to the Poisson point process X . Hence, assumption $i)^\dagger$ is effectively indispensable since the distribution of $L(t)$ is generally intractable if we consider a more complicated type of point process. The key feature of assumption $ii)^\dagger$ is expressed in (8), namely that each remaining lifespan has an exponential distribution no matter the current age of the considered path component (inherited from the memoryless property). If an alternative lifetime distribution is employed the remaining lifespan of each “active” path component will inevitably depend on age.

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